Cluster emission in intermediate-energy nuclear reactions and the nuclear “liquid-gas phase transition”

Basic goal: Quantal A body system

Lack of knowledge & understanding

→ Beyond the mean field, correlations
→ Nuclear stability at extremes (A, Z, E*, I)
→ Nuclear thermodynamics/transport
→ Isospin (asymmetry) effects
→ Role of the diffuse nuclear surface

Accessibility in (peripheral) heavy-ion collisions at intermediate energies, (perhaps also in fusion)
Outline

• Introduction: Intermediate energy HI reaction domain
  How to map nuclear phase diagram (?)

• Experiment: Cluster emission
  in the dissipative reaction environment

• Models: Conventional
  Expansion and surface phenomena,
  stability and decay of hot nuclei

• Conclusions
Macroscopic clusters: Surface ablation with lasers

(Zhigelej et al., 2000)

Goals:

Mechanism (thermodynamical, dynamical),
dependence of cluster multiplicities & sizes on energy
→ surface physics, nano materials

<table>
<thead>
<tr>
<th>Low energy</th>
<th>High energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser ablation in the regime of thermal confinement</td>
<td></td>
</tr>
<tr>
<td>Fluence is right above the threshold for the onset of ablation</td>
<td></td>
</tr>
<tr>
<td>Laser pulse duration is 150 ps, penetration depth is 50 nm</td>
<td></td>
</tr>
</tbody>
</table>

Low energy: few large clusters
High energy: many small clusters → vaporization

Equilibrium molecular dynamics models
Heavy-Ion Reaction Scenarios

Near the Barrier
\((E/A = 5-20 \text{ MeV})\)

Dissipative Collisions leading to Focussing and Orbiting

2 possible emitters: PLF, TLF

Fermi/Intermediate Energies
\((E/A = 20-100 \text{ MeV})\)

Peripheral
Participant-Spectator Scenario (Fireball)
3 emitters: PLF, TLF, IVS

Fermi/Intermediate Energies: Central
Multi-Fragmentation (Fireball at high energies)
1 emitter: CN
Interaction and Relaxation Times Scales

Thermal relaxation times \((BUU)\) decrease with \(T\).
Slower relaxation at higher density \(\rho\).

\[
t_{\text{rel}}(T, \rho) = \frac{310}{T^2} \left( \frac{\rho}{\rho_0} \right)^{0.4} \left( 1 + 0.1T \left( \frac{\rho}{\rho_0} \right) \right) + \frac{57T^{-1/2}}{1 + 160/T^2} \left( \frac{\rho}{\rho_0} \right)
\]

\(t_{\text{evap}} \rightarrow \) experimental CN systematics.

\(t_{\text{evap}}(T) \approx 5.6 \cdot 10^{-24} e^{13\text{MeV}/T} \text{s}\)

\(t_{\text{rxn}} \rightarrow \) dynamical NEM simulations \(Bi+Xe\)

Limits to heat generation in nuclei
Thermal stability: \(t < t_{\text{rxn}}\) interaction/acc time

Evap too fast for fission!
Outline

• Introduction: Intermediate energy HI reaction domain
  How to map nuclear phase diagram (?)

• Experiment: Cluster emission in the dissipative reaction environment

• Models: Conventional Expansion and surface phenomena, stability and decay of hot nuclei

• Conclusions
Bulk Properties: The Nuclear Equation of State

(after Bertsch & Siemens (PLT 126B,9): Skyrme interaction)

Spinodal (for uniform $\rho$):

Compressibility $\kappa^{-1} = \rho \cdot \frac{\partial p}{\partial \rho} = \rho^2 \cdot \frac{\partial^2 f}{\partial \rho^2}$

Unstable modes $k < k_{\text{max}} = \left(-\kappa \rho^2 c\right)^{-1/2} \rightarrow \rho_k \sim e^{\omega_k t} \rightarrow \text{exponential growth}$

$k$: wave number, $\omega_k$: frequency
Mechanical Nuclear Instabilities

Density multipoles, sharp sphere (bulk), high E* (schematic treatment, surface, Coulomb barrier)

Doorway states: assumptions

Isentropic expansion $\rightarrow$ fragmentation

Has to overcome 3-body saddle

Isentropic? No entropy generation? No dissipation? $\rightarrow$ Calls for experimental test

Cluster decay = signature of spinodal decomposition?
How to Map the Nuclear Phase Diagram

Phases of infinite nuclear matter

Attempt to reach unstable (spinodal) region with nuclear reactions → cluster decay?

How to produce an equilibrium situation (heat bath?)
Limits to Nuclear Heating

Limiting temperatures $T_{\text{lim}}$ from sequential particles $T_{\text{lim}}$ cannot be exceeded.

$T > 7 \text{ MeV}$: resolution problems (PE)

Negative heat capacities?

Potential reasons for limiting $T$:

- small $t_{\text{evap}}$
- preequilibrium emission
- expansion cooling
Decay via Intermediate-Mass Clusters (?)

\[ m_{\text{IMF}} = \text{multiplicity of IMF clusters} \]

\[ \langle Z_{\text{cluster}} \rangle \approx 8 \text{(oxygen)} \]
\[ \langle E_{\text{cluster}} \rangle \approx 25 - 30 \text{MeV} \]

- Statistical independence of cluster emission

Projectile dissipates 2 MeV/nucleon for each cluster.
Outline

• Introduction: Intermediate energy HI reaction domain
  How to map nuclear phase diagram (?)

• Experiment: Cluster emission
  in the dissipative reaction environment

• Models: Conventional
  Expansion and surface phenomena,
  stability and decay of hot nuclei

• Conclusions
Some Experimental Data

Expt. collaboration:

J. Tõke, W. Gawlikowicz, J. Lu, WUS (Rochester)
R.G. Charity, L.G. Sobotka (St. Louis),
R.T. deSouza (Bloomington)

Examples:

Phenomenology of the HI reactions

\[ ^{209}\text{Bi} + ^{136}\text{Xe} \ (\varepsilon_{\text{Lab}} = 7, \ldots, 62 \text{ MeV}) \]

\[ ^{197}\text{Au} + ^{86}\text{Kr} \ (\varepsilon_{\text{Lab}} = \ldots, 35, 42, 54 \text{ MeV}) \]

\[ : \]

Survey experiments
SuperBall/Dwarf Calorimeter

Experiments at NSCL/MSU

$4\pi$ measurement of neutrons + LCPs $\rightarrow$ excitation energy, impact parameter

$4\pi$ measurement of charged particles, sampling of massive PLF/TLF
Clusters & LGPT

M_2006 W. Udo Schröder

Dissipative Reaction Environment PLF Distributions

\[ ^{209}\text{Bi} + ^{136}\text{Xe} \]

28 MeV/nucleon

62 MeV/nucleon

2 massive fragments survive PLF + TLF

\[ \text{Beam Direction} \]

\[ \Theta_{\text{proj}} \text{[deg]} \]

\[ \frac{E_{\text{plf}}}{E_{\text{beam}}} \]

NEM: Stable \( <Z_{\text{PLF}}> \approx Z_{\text{proj}} \)

PLF remnant distributions from evaporative decay

\[ \rightarrow <Z_{\text{PLF}}>, <\sigma^2_{Z_{\text{PLF}}}> \]
Thermal Observables: Joint Multiplicity Distributions

$^{209}\text{Bi}+^{136}\text{Xe}$, $E_{\text{Lab}}/A=62$ MeV

$P(m_n, m_{\text{LCP}})$ \text{(log contours)}

Dotted: average ridge lines for $E/A=28, 40, 62$ MeV.

$\rightarrow \rightarrow E^*, b$

Independent of $E_{\text{Lab}}$

$P(E^*) = \tilde{P}[P(m_n, m_{\text{LCP}})]$

$\neq f(E_{\text{lab}})!!$

$P'(b) = \tilde{P}'[P(m_n, m_{\text{LCP}})]$

\text{Also (SB): } P(TKE_n), 5 \Delta\Theta
Unusual Statistical Features of Cluster Emission

Regions in $m_n/m_{lcp}$ plane correspond to regions in total excitation energy $E^*$ of reaction fragments.

For $m_{IMF} = 3, 4..., 8$ no change in illuminated $E^*$ region.

Challenge: statistical competition clusters and light particles?

→ spontaneous multi-fragmentation?
Detector Response to Charged Particles

Average cluster emission patterns ($Z_{\text{cluster}} > 2$, deduce $A$ from $A \approx A_{\text{attractor}}$)

Energy resolution, angular resolution (granularity), detection/ID thresholds all distort the ideal (circular) kinematical pattern.
Light Charged Particles and Clusters

Sources are kinematically correlated with
PLF (projectile-like fragment)
TLF (target-like fragment)
IVS (Intermediate velocity source = “CN”, pre-equilibrium)

Similar invariant velocity distributions for
LCP & IMF clusters (3 ≤ Z ≤ 10)

Concentrate first on sequential cluster emission from PLF (well prepared)
Moving-Source Parameterization of Cluster Emission

Cluster multiplicities increase with decreasing b, increasing \( m_{IVS}/(m_{PLF}+m_{TLF}) \). IVS clusters have much harder ("PE" type) spectra than PLF clusters.
Outline

• Introduction: Intermediate energy HI reaction domain
  How to map nuclear phase diagram (?)

• Experiment: Cluster emission in the dissipative reaction environment

• Models: Conventional Expansion and surface phenomena, stability and decay of hot nuclei

• Conclusions
Challenges to Conventional Statistical Models

Fundamental problems for stat. cluster emission ($m_{\text{cluster}} > 1$):

- high emission barriers: $V_B \approx (50-60) \text{ MeV}$, $T \leq 5-6 \text{ MeV}$
- nuclear interactions non negligible even at $\rho < \rho_0/3$.

**Shortcomings of conventional statistical models**

→ No, or unrealistic, reaction scenarios

**EESM:** “scales” the barrier at the expense of compressional, and not thermal energy.

**SMM, MMMC:** “scale” the barrier one nucleon at time – rely on randomized gas flux (perfectly reflecting container walls). limited account of entropy (neglect of $\rho < \rho_0$, level density)

**Fisher’s droplet model:** does not treat emission. Questionable adaptations

**Percolation:** does not treat emission.
Number (~ a ) of states/nucleon is larger in the diffuse surface than in the dense bulk.
Tõke & Swiatecki NPA 372, 141(1981)

Fermi gas density of states

\[ \Omega(A,E^*) = e^{2\sqrt{a(A)E^*}} = e^{S(A,E^*)/k} \]
Thermal Expansion: Equilibrium Density

\[ E^* = E_{compr}^* + E_{therm}^* = E_{bind}(1 - \frac{\rho}{\rho_0})^2 + a_0 \cdot \left( \frac{\rho}{\rho_0} \right)^{-2/3} T^2 \]

EOS-harmonic approximation

**Level Density Parameter**

\[ a = a_{volume} + a_{surface} \]

\[ = (A\alpha_v + A^3\alpha_s)(\rho/\rho_0)^{-2/3} \]

\[ S = 2\sqrt{a \cdot E_{therm}^*} \]

\[ (\partial S/\partial \rho)_{E^*} = 0 \rightarrow \]

\[ \frac{\rho_{eq}}{\rho_0} = \frac{1}{4}(1 + \sqrt{9 - 8 \frac{E_{Total}}{E_{Binding}}}) \]

Density drops to \( \rho_0/3 \) before disassembly \( \rightarrow \) nucleons
Effects of Irreversible Nuclear Expansion

Caloric Curve

T non monotonic, plateau

(Effects of $m^*$)
Sobotka PRL 2004

Softening/melting of surface $\rightarrow$ reduction in energy of surface normal modes $\rightarrow$ stability loss.

Evaporation time

Reduction of $T$
$\rightarrow$ increased evaporation time
$\rightarrow$ increased stability/lifetime
Multipole Surface Vibrations as MF Doorways?

\[ C_\lambda = (\lambda - 1)(\lambda + 2) \frac{b_s}{4\pi} A^{2/3} - \frac{5}{2\pi} \frac{\lambda - 1}{(2\lambda - 1)} b_c \frac{Z^2}{A^{1/3}} \]

Bohr & Mottelson II, Ch. 6

\[ \longleftrightarrow \text{ actually an over-estimate!} \]

\[ b_s(T) = 17.5 \text{ MeV} \cdot \left( \frac{T_c^2 - T^2}{T_c^2 + T^2} \right)^{5/4} \]

\[ b_c(T) = 0.7 \text{ MeV} \cdot \left(1 - 7.6 \cdot 10^{-4} \left( \frac{T}{\text{MeV}} \right)^2 \right) \]

With increasing \( E^* \)

→ decreasing \( \gamma \)
→ decreasing energy of \( \lambda \) mode
→ \( \sim \)-degeneracy of different \( \lambda \)
→ develop at same \( E^*(m_n, m_{lcp}) \)

\[ \lambda = 8 \]
\[ \lambda = 6 \]
\[ \lambda = 4 \]
\[ \lambda = 2 \]
Cluster Decay of Excited Nuclei

Entropy per nucleon for

- **Mono-nucleus:** $^{208}$Pb* parent
- **Dinucleus:** $^{192}$W residue in nuclear contact with $^{16}$O cluster

→ True saddle-point

Potential Energy of Test Particle W+O
Effective Emission Barriers

\[ P \propto e^{\Delta S} = e^{-\frac{B_{\text{Eff}}}{T}} \quad B_{\text{Eff}} = -T \Delta S \quad \Delta S = S_{\text{Saddle,diN}} - S_{\text{monoN}} \]

Dramatic entropy gains at high T favor cluster emission.

Without surface effects

With surface effects

\[ B_{\text{Eff}} \text{ (MeV)} \]

\[ T \text{ (MeV)} \]
For $E^*/A > 5$ MeV, $^{16}$O emission from $^{197}$Au more likely than proton emission!

$m_{\text{IMF}}>1$ because $Q<0$ for heavy CN*
Academic Considerations: Boxed Fermi Matter

If one could confine nuclear matter \((A, Z)\) in a box with perfectly reflecting walls \((V = V_{\text{liqu}} + V_{\text{gas}} = \text{const.}, \ T = \text{const.})\) → equilibration to maximum entropy → minimum of free energy \(F\)

\[
F = E_{\text{total}}^* - T \cdot S = E_{\text{compr}}^* + E_{\text{therm}}^* - 2aT^2
\]

\[
= E_{\text{compr}}^* - aT^2 = E_{\text{bind}}(1 - \rho/\rho_0)^2 - a_0 \cdot (\rho/\rho_0)^{-2/3}T^2
\]

\[
F_{\text{gas}} = A_{\text{gas}} \varepsilon_{\text{Bind}} \left(1 - \frac{A_{\text{gas}}}{V_{\text{gas}} \rho_0}\right)^2 - A_{\text{gas}} a_0 \left(\frac{A_{\text{gas}}}{V_{\text{gas}} \rho_0}\right)^{-2/3} T^2
\]

\[
F_{\text{liqu}} = A_{\text{liqu}} \varepsilon_{\text{Bind}} \left(1 - \frac{A_{\text{liqu}}}{V_{\text{liqu}} \rho_0}\right)^2 - A_{\text{liqu}} a_0 \left(\frac{A_{\text{liqu}}}{V_{\text{liqu}} \rho_0}\right)^{-2/3} T^2
\]
Isotherms of Confined Nuclear Fermi Matter

Real gas of interacting nucleons in confined volume $V$, $T$-const.

Compression leads to liquefaction. Results automatically from minimization of

$$ F = F_{\text{liqu}} + F_{\text{gas}} $$

Maxwell construction not necessary.
Critical exponent $\alpha = 0.5$ for $\rho(T-T_c)$

$T_c = 10.02 \text{ MeV}$

$V_c = 2.4 V_o$

$r_c = 0.417 r_o$
Conclusions: Dynamics & Thermodynamics of Cluster Decay

- There are different modes of cluster emission: non-equilibrium and equilibrium

- Found that expansion of hot nuclear matter changes high-$T$ statistical decay pattern significantly $\Delta S$, $\Delta S_{\text{surf}}$

- Entropy-driven decay favors clusters over simple particles, surface melting equivalent to phase transition

- Exact mechanism of cluster formation and decay not yet determined.

- If one can establish mechanism, then new access to (isospin) nuclear EOS.

- New important study of nuclear surface becomes possible
The End