Dynamics and Thermodynamics in Nuclear Heavy-Ion Collisions

I. Introduction: Testing the nuclear A-body system in heavy-ion collisions

II. Experimental Techniques

III. Dynamics of heavy-ion collisions at medium energies

IV. Thermodynamics of hot nuclear fragments

Collaboration
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Isospin-Dependent In-Medium Effects

Mean Field (Energy)

NN Interaction

Residual Interactions

density dependent

NN Correlations Fluctuations

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Isospin-Dependent Nucleus-Nucleus Interactions

Isovector Proximity Potential FF

Density Distributions

2-Body Friction FF

Coherent, Diabatic Emission of Nucleons

Stochastic Emission of Preequil.-Cascade Nucleons

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Characteristic Times and Lengths

Velocity of sound:
\[ v_s = 0.2 \, c = 6 \, \text{fm}/(10^{-22} \, \text{sec}) \]

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Intrinsic degrees of freedom:

\[ t_{\text{relax}} \approx 3.3 \times 10^{-20} \text{s} / E_p (\text{MeV}) \]
\[ \sim 10^{-22} \text{s} \]

Collective shape degrees of freedom:

\[ t_{\text{deform}} \sim 10^{-21} \text{s} \text{ (reactions)} \]
\[ \sim 10^{-19} \text{s} \text{ (fission)} \]
Equilibration vs. Decay

Thermal relaxation times (BUU) decrease with T. Slower relaxation at higher density $\rho$.

Particle evaporation times from experimental CN systematics. Uncertain $\sim 10$.

Limits to heat generation in nuclei
Experimental Methods

Highly excited reaction products \(\rightarrow\) measure as many products as possible with broad dynamic range:

- neutrons, p, d, t ..., IMF clusters, PLF, TLF over entire angular range \((\Omega \approx 4\pi)\)

Develop observables measuring \(E^*\) (kinetic E-loss)

\(\rightarrow\) impact parameter/”violence” of collision
SuperBall/Dwarf Calorimeter

$4\pi$ measurement of neutrons $\rightarrow$ excitation energy, impact parameter

$4\pi$ measurement of charged particles

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Thermal Observables: Joint Multiplicity Distributions

Probability distribution of neutrons and lcp's from the reaction $^{209}\text{Bi} + ^{136}\text{Xe}$ at $E/A=62$ MeV (log contour diagram).

Dotted: average ridge lines for $E/A=28$, 40, 62 MeV.

$\rightarrow \rightarrow E^*, \ b$

\[
P(E^*) = \tilde{P}\left[P\left(m_n,m_{LCP}\right)\right]
\]

\[
P'(b) = \tilde{P}'\left[P\left(m_n,m_{LCP}\right)\right]
\]

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Random emission leads to a spherically symmetric emission patterns in velocity space.

→ Plot Galilei-invariant cross sections.
The Experimental Filter

Energy resolution, angular resolution (granularity), detection/ID thresholds all distort the ideal kinematical (circular) pattern
Nuclear Dynamics

- Gross properties of nuclear reactions (coupling of intrinsic to macroscopic variables) → Transport models (NEM, BUU/BNV, QMD,...)

- deflection functions, dissipation and mass transfer, equilibration

- Fast (non-equilibrium) processes → nucleon and cluster emission (breakup/fracture)

- Slow, sequential (equilibrium) processes → nucleon and cluster evaporation
Sensitivity of Deflection Functions

Standard dynamical NEM calculation. Observables

\[ \tilde{x} = \{x_i\} = \{A_{PLF}, Z_{PLF}, r, \ell, \text{shape}\} \]

t-dependent joint probability distribution \( P(\tilde{x}, t) \) for macroscopic observables:

**Fokker-Planck Equation**

\[
\frac{\partial}{\partial t} P(\tilde{x}, t) = -\sum_i \frac{\partial}{\partial x_i} \{v_i P(\tilde{x}, t)\} \\
+ \sum_{i,j} \frac{\partial^2}{\partial x_i \partial x_j} \{D_{i,j} P(\tilde{x}, t)\}
\]

**Lagrange-Rayleigh Equ.**

\[
\left\{ \frac{d}{dt} \frac{\partial}{\partial \tilde{x}_i} - \frac{\partial}{\partial \tilde{x}_i} \right\} \mathbf{L} = -\frac{\partial}{\partial \tilde{x}_i} \mathbf{F}
\]
PLF Deflection Functions/E-Z Distributions

28 MeV/nucleon  62 MeV/nucleon  $^{209}\text{Bi} + ^{136}\text{Xe}$

NEM:
Stable $\langle Z_{\text{PLF}} \rangle \approx Z_{\text{proj}}$

PLF remnant distributions from evaporative decay
Add $Z$ of evaporated particles (LCP) to $Z$ of emitter remnant. Feasible for mean values $<Z>$. 

Stable $<Z> \approx Z_{\text{proj}}$. Decrease by IMF cluster emission.
Equilibrium and Non-Equilibrium Cluster Emission

2 cluster emission mechanisms: Sequential from PLF and TLF plus hard pre-equilibrium spectrum $E_0 \gg T$ (3rd source)

Typical cluster: $^{16}\text{O}$ with thermal spectrum

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Equilibrium and Non-Equilibrium Nucleon Emission

$^{197}$Au+$^{208}$Pb, E/A=29 MeV

$v_{\text{beam}} = 7.5 \text{ cm/ns}$

Triple kinem. coincidence: PLF-TLF-neutron.

$T \approx 2 \text{ MeV}$,

$m_n(\text{PLF}) \approx m_n(\text{TLF}) \approx 4$

- PLF or TLF evaporation
- Non-equilibrium (Fermi jets?), high energy neutrons
Understanding the Dynamics (?)

• Qualitative description of some systematic aspects possible with Nucleon Exchange Model, also BUU, BNV, etc.

• But no consistent modeling of significant reaction features (e.g., fast particle and cluster emission) achieved as yet.

• Have identified important trends, probably similar physics and transition to new phenomena at E/A = 50-100 MeV.
Thermodynamics of Nuclear Decay

- Approach to thermal and chemical equilibrium
- Consequences of the equation of state of hot nuclear matter
- Limiting temperatures, apparent heat capacities
- Isospin dependence
- (Academic considerations)

No satisfactory explanation is offered by existing statistical models on:
- massive fragment emission vs. proton and LCP emission
- Limiting temperatures, unusual (negative) heat capacities?
Limiting Excitations/Temperatures?

\[ ^{197}\text{Au} + ^{86}\text{Kr} \quad E/A = 39,55 \text{ MeV} \]

The \( M_n - M_{\text{lcp}} \) distribution should reflect \( E^* \) distribution.

**Puzzle:** Why do multiplicities not increase \( \propto \frac{E}{A} \)?

Where does energy go?

\[ \rightarrow \text{IMF-cluster emission} \]
Regions in $m_n/m_{lcp}$ plane correspond to regions in excitation energy of reaction fragments.

For $m_{\text{IMF}} = 3, 4, ..., 12$, no change in illuminated $E^*$ region.

Difficult to understand: statistical competition clusters and light particles.

→ spontaneous multi-fragmentation?
Alternative: Entropy-Driven Cluster Emission

\[ S = S_{\text{mono}N} \]

\[ S_{\text{Saddle,diN}} = S_{\text{mono}N} + \Delta S \]

Weisskopf emission probability

\[ p \propto e^{\Delta S} = e^{-\frac{B_{\text{Eff}}}{T}} \]

Effective barrier for emission

\[ B_{\text{Eff}} = -T \Delta S \]

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Interacting Fermi Matter (Harmonic Approximation)

\[ E_{\text{Total}}^* = E_{\text{Compressional}}^* + E_{\text{Thermal}}^* \]

\[ E_{\text{Compressional}}^* = -E_{\text{Binding}} \left(1 - \frac{\rho}{\rho_o}\right)^2 \]

\[ E_{\text{Thermal}}^* = a T^2 \quad | \text{Level density parameter } a = a_o \left(\frac{\rho}{\rho_o}\right)^{-\frac{2}{3}} \]

Entropy:
\[ S = 2 \sqrt{a E_{\text{Thermal}}^*} = 2 \sqrt{a (E_{\text{Total}}^* - E_{\text{Compressional}}^*)} \]

Thermodynamic Equilibrium:
\[ \frac{\partial S}{\partial \rho} = 0 \rightarrow \rho_{\text{Equil}} \]

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Equilibrium Density of Interacting Fermi Matter

Equilibrium Nuclear Density

\[
\frac{\rho_{eq}}{\rho_o} = \frac{1}{4} \left( 1 + \sqrt{9 - 8 \frac{E_{Total}}{E_{Binding}}} \right)
\]

\[ S = 2 \sqrt{a \cdot E_{therm}} \]

Level Density Parameter

\[
a = a_{Volume} + a_{Surface}
\]

\[ = (A \alpha_V + A^3 \alpha_S) \left( \frac{\rho}{\rho_o} \right)^{-\frac{2}{3}} \]

Density drops to 25% before disassembly

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Caloric Curve for the Fermi Matter

\[
T = \sqrt{\frac{E_{\text{Total}}^*}{a}} = \left(\frac{\rho_{\text{Equil}}}{\rho_o}\right)^{-\frac{1}{3}} \frac{1}{\sqrt{a_o}} \sqrt{E_{\text{Total}}^* - E_{\text{Bind}}(1 - \frac{\rho_{\text{Equil}}}{\rho_o})^2}
\]

Non-monotonic relation \(E_{\text{total}}^* \leftrightarrow T\)

Limiting temperature: nucleus disassembles into nucleons

Depends on \(E_{\text{Bind}}\) (isospin dependence)

Apparent heat capacity can be \(C < 0\)

A = 208
Cluster Decay of Excited Nuclei

Entropy per nucleon for
- $^{208}\text{Pb}^{*}$ parent mono-nucleus
- Transition state: $^{192}\text{W}$ residue in nuclear contact with $^{16}\text{O}$ cluster

→ True saddle-point

Entropy $S/A$ vs. total excitation $E^*/A$

Potential Energy of Test Particle $\text{W}+\text{O}$

$V_{\text{Coul}}$, $V_{\text{Nuclear}}$, $V_{\text{Total}}$ vs. distance from residue (fm)
Effective Emission Barriers

Dramatic entropy gains at high $T$ favor cluster emission
For $E^*/A > 5$ MeV, the emission of $^{16}$O from $^{197}$Au is more likely than proton emission!
Some Academic Considerations

If one could confine nuclear matter \((A, Z)\) in a box with perfectly reflecting walls \((V=V_{\text{liqu}}+V_{\text{gas}}=\text{const.}, T=\text{const.})\) → equilibration to maximum entropy

\[
F = E^*_{\text{total}} - T \cdot S = E^*_{\text{compr}} + E^*_{\text{therm}} - 2aT^2
\]

\[
E^*_{\text{compr}} - aT^2 = E_{\text{bind}}(1 - \rho/\rho_0)^2 - a_0(\rho/\rho_0)^{-2/3}T^2
\]

\[
F_{\text{gas}} = A_{\text{gas}} \varepsilon_{\text{Bind}} \left(1 - \frac{A_{\text{gas}}}{V_{\text{gas}} \rho_0}\right)^2 - A_{\text{gas}} a_0 \left(\frac{A_{\text{gas}}}{V_{\text{gas}} \rho_0}\right)^{-2/3} T^2
\]

\[
F_{\text{liqu}} = A_{\text{liqu}} \varepsilon_{\text{Bind}} \left(1 - \frac{A_{\text{liqu}}}{V_{\text{liqu}} \rho_0}\right)^2 - A_{\text{liqu}} a_0 \left(\frac{A_{\text{liqu}}}{V_{\text{liqu}} \rho_0}\right)^{-2/3} T^2
\]

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Isotherms of Confined Nuclear Fermi Matter

Real gas of interacting nucleons in confined volume $V$, $T$-const.

Compression leads to liquefaction. Results automatically from minimization of $F = F_{\text{liqu}} + F_{\text{gas}}$.

Maxwell construction not necessary.
Liquid-Gas Coexistence Line of Fermi Matter

Critical exponent $\alpha = 0.5$ for $\rho(T-T_c)$
Understanding the Thermodynamics (?)

- Found that expansion of hot nuclear matter changes decay pattern significantly $\Delta S$, $\Delta S_{\text{surf}}$

- Entropy-driven decay favors clusters over simple particles

- Exact mechanism of cluster decay not determined as yet.

- If one can establish mechanism, then new access to (isospin) nuclear EOS.